Effect of Normalization on TOPSIS and Fuzzy TOPSIS

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Abstract

In a recent study, the authors evaluated five multi-criteria decision-making methods and have shown inconsistent results of Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and fuzzy TOPSIS compared to Analytic Hierarchy Process (AHP), fuzzy AHP, and Preference Ranking Organization METHod for Enrichment Evaluations (PROMTHEE). Several studies evaluated different normalization techniques for TOPSIS and justified the use of vector normalization. However, this study shows that the vector normalization method used in TOPSIS and a local normalization method used in fuzzy TOPSIS to construct a normalized fuzzy decision matrix can lead to inconsistent ranking results of TOPSIS and fuzzy TOPSIS compared to AHP and PROMETHEE. Some improvements are suggested that result in identical ranking results among AHP, PROMETHEE, and TOPSIS. Similarly, the results of fuzzy TOPSIS can be improved by fixing its normalization step.

Keywords: TOPSIS, fuzzy TOPSIS, vector normalization, reciprocal normalization, Multi-criteria decision-making methods.

1. INTRODUCTION

Multi-criteria decision-making (MCDM) methods are used to help decision-makers to evaluate and rank a finite number of alternatives with respect to multiple competing criteria. For instance, to buy a car, MCDM methods evaluate criteria such as quality, style, reliability, performance, and price of the available car models to choose the best compromise alternative car. Numerous MCDM methods have been invented, such as Analytic Hierarchy Process (AHP) (Saaty, 1980), Analytic Network Process (ANP) (Saaty, 1996), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) (Hwang and Yoon, 1981), Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE) (Brans and Vincke, 1985), ELimination and Choice Expressing the Reality (ELECTREE) (Roy, 1991), and Viekriterijumsko Kompromisno Rangiranje (VIKOR) (Opricovic, 1998).

Human judgment can be vague and imprecise; therefore, fuzzy multi-criteria decision-making methods (FMCDM) should be used to replace the exact values and handle vagueness and uncertainty. Fuzzy logic is first introduced by Zadeh (1965) to deal with uncertainties and imprecise inputs. Fuzzy AHP and fuzzy TOPSIS are the most and the second most widely used fuzzy MCDM techniques from 1980 to 2014 (Kahraman, Onar, and Oztaysi, 2015).

Multi-criteria decision analysis methods have been widely used in many areas, including economics, education systems, environment, finance, politics, health care, and transportation (Shim, 1989). For instance, in the higher education industry, MCDM methods have been mostly used for resource allocation, followed by planning and evaluation (Mustafa and Goh, 1996). Ding & Zeng (2015) used TOPSIS combined with information entropy weight (IEW), to investigate the performance of 68 Chinese universities belonging to the Ministry of Education (MOE) from 2002 to 2011. Badri and Abdulla (2004) used AHP to determine the performance of higher education faculty members in terms of research, teaching, and university and community service. In the Transportation field, Saaty (1995) showed five examples of using AHP in transportation and illustrated the use of AHP in choosing the best route to commute from Saaty's home to the University of Pennsylvania. Yang (2010) presented an enhanced emergency routing method for fire forces based on Dijkstra's algorithm and AHP. Bao, Ruan, Shen, Hermans, and Janssens (2012) incorporated fuzzy TOPSIS to evaluate the road safety performance of 21 European countries.

The authors previously presented a comparative study of five multi-criteria decision-making methods, namely AHP, Fuzzy AHP, TOPSIS, Fuzzy TOPSIS, and PROMETHEE through two real-world case studies (Sarraf & McGuire, 2020). AHP was chosen due to its simplicity, the ability to handle both qualitative and quantitative data, the ability to derive criteria weights, and above all it is the most widely used method in the literature. TOPSIS was chosen because it represents human rationale in judgement and tries to minimize the distance to the best solution and maximize the distance from the worst solution. PROMETHEE was chosen since it is based on pairwise comparison that comes naturally and it can give a full ranking of alternatives. Fuzzy AHP and fuzzy TOPSIS were chosen because they can handle vagueness and uncertainty, and it is the most and second most widely used fuzzy MCDM techniques.

Sarraf and McGuire (2020) applied the multi-criteria decision-making methods on five alternative paths from a given source to a given destination to rank them based on a user's input. The selected criteria were travel time, travel distance, and safety level. The safety level of roads was calculated based on historical crash data on each path. The study showed that the ranking results of PROMETHEE fit well with the ranking results of AHP, and they both result in the best ranking. Fuzzy AHP also produced good ranking results. On the other hand, TOPSIS and fuzzy TOPSIS produced inconsistent and poor ranking results compared to AHP and PROMETHEE. Normalization is a crucial step of MCDM methods to transform the measurements in the matrix of alternatives into dimensionless and comparable values. The normalization step can have a direct effect on the results of decision-making methods. In this research study, the inconsistent results of TOPSIS and fuzzy TOPSIS are investigated, and suggestions are made to improve these methods.

This research paper is organized as follows. The literature review section presents some research studies evaluating the effects of different normalization methods on TOPSIS. Section 3 presents details of TOPSIS. Section 4 presents details of Chen's(2000) fuzzy TOPSIS. Section 5 presents the initial results of the case studies. Section 6 presents the suggestions to improve the results of TOPSIS and fuzzy TOPSIS. Section 7 presents the concluding remarks.

2. LITERATURE REVIEW

Different normalization methods may be used in multi-criteria decision-making techniques. There have been several studies evaluating the influence of normalization on the ranking results of MCDM methods, especially on TOPSIS. Although the normalization process scales the criteria values to be approximate of the same magnitude, different normalization methods may result in different solutions (Chatterjee & Chakraborty, 2014).

Chakraborty and Yeh (2009) analyzed the effect of vector normalization and three linear scale transformations (max-min, max, and sum) on TOPSIS. The results justified the use of the vector normalization procedure for TOPSIS. However, the study shows that as the problem size increases from 4 attributes and 4 alternatives to 20 attributes and 20 alternatives, the ranking consistency of all four normalizations drops significantly. In addition, a diverse range of data (1-10, 1-100, 1-500, 1-1000, 1-2500, 1-5000, 1-7500, 1-10000) for each criterion was chosen to analyze the ranking consistency for various data range (wide range, moderate range, and narrow range). The results are rated as best, good, average, and poor. Although the vector normalization method outperformed other normalization techniques over all data ranges, the sensitivity to weights of criteria revealed that for the wide range category, none of the normalization methods could be rated as best method. For the wide range category, the vector normalization was rated as good, while the max-min, max and sum normalization methods were rated poor, good, and average, respectively.

Chatterjee and Chakraborty (2014) investigated the results of different normalization methods on the PROMETHEE, TOPSIS, and GRA techniques. For flexible manufacturing system selection, four different normalization methods, namely, vector normalization (VN), Weitendorf's linear normalization (WLN), Jüttler's-Körth's normalization (JKN), and Non-linear normalization (NLN) were used. The authors presented the ranking performance for the three MCDM methods with respect to the four different normalization methods. Spearman's rank correlation coefficient (r_s) were calculated to determine the rank agreement between two sets of rankings. The Average rs value between these MCDM methods were also computed to determine the mean ranking agreement among themselves. The results clearly showed that PROMETHEE II with the highest mean r_s value of 0.9167 is less affected by different normalization methods while TOPSIS with the least mean r_s value of 0.5654 is the most sensitive MCDM method to the normalization methods. In addition, Vector normalization with the highest r_s value of 0.9762 is the most preferred method and JKN with the least r_s value of 0.3333 is the least preferred normalization method.

Vafaei et al. (2018) evaluated the results of six normalization methods, linear (max), linear (max-min), linear (sum), vector normalization, logarithmic normalization, and fuzzification on TOPSIS, and concluded that vector normalization is the best normalization technique for TOPSIS and logarithmic normalization technique is the worst one. Similarly, Celen (2014) evaluated the effects of vector normalization and three linear normalization methods (max-min, max, and sum) on TOPSIS and concluded that vector normalization, which is the default normalization procedure for TOPSIS, generated the most consistent results.

Pavlicic (2001) analyzed the effects of simple linear and vector normalization techniques on the results of TOPSIS, ELECTRE, and Simple Additive Weight (SAW). The results of MADM methods when using vector normalization or simple normalization could depend on measurement units. Pavlicic suggested a call for reconsideration of the use of some normalization methods used in MCDM methods including vector normalization.

Milani et al. (2005) studied the effect of eight normalization methods in TOPSIS while applied to gear material selection for a power transmission problem. They concluded that different normalization procedures generated rather different closeness coefficients. Based on the results, it was verified that linear optimization norms cannot affect the rank of alternatives significantly, while nonlinear norms may yield some deviations.

3. TOPSIS

TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) is a Multi-Criteria Decision Making Method that was first developed by Hwang and Yoon (1981). TOPSIS is based on an aggregation function to rank alternatives and to determine the best alternative among a finite set of alternatives. TOPSIS works based on minimization of Euclidean distance from the positive ideal solution and maximization of Euclidean distance from the negative ideal solution. The TOPSIS procedure is based on the following steps:

Step 1: Given matrix $A(x)_{mn}$ Calculate the normalized decision matrix as shown in equation (1). This is the default vector normalization used in TOPSIS.

$$r_{ij}(x) = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, i = 1, \dots, m;\ \ j = 1, \dots, n. \quad (1)$$

Step 2: Based on the weights of criteria, calculate the weighted normalized decision matrix as shown in equation (2).

$$v_{ij} = w_j\, r_{ij}\ ,\ \ i = 1, \dots, m;\ \ j = 1, \dots, n. \quad (2)$$

where w_j is the weight of the jth criterion.

Step 3: Determine the positive ideal and negative ideal solutions as shown in equation (3).

$$A^+ = \{v_1^+, v_2^+, \dots, v_n^+\}$$
where $v_j^+ = \{\max(v_{ij})$ if $j \in B$; $\min(v_{ij})$ if $j \in C\}$

$$A^- = \{v_1^-, v_2^-, \dots, v_n^-\} \quad (3)$$
where $v_j^- = \{\min(v_{ij})$ if $j \in B$; $\max(v_{ij})$ if $j \in C\}$

where B is associated with the benefit and C is associated with the cost criteria.

Step 4: Based on equation (4), calculate the separation from positive ideal and negative ideal solutions for each alternative.

$$S_i^+ = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^+)^2}\ , i = 1, \dots, m$$
$$S_i^- = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^-)^2}\ , i = 1, \dots, m \quad (4)$$

Step 5: Calculate the relative closeness to the ideal solution as shown in equation (5).

$$C_i^- = \frac{S_i^-}{S_i^+ + S_i^-} \quad , i = 1, \dots, m \quad (5)$$

Finally, rank the alternatives based on C_i^-.

4. FUZZY TOPSIS

There are several fuzzy TOPSIS methods developed to address the vagueness of human judgment (Abo-Sinna & Abou-El-Enien, 2005; Chen & Hwang, 1992; Chen, 2000; Liang, 1999; Wang & Elhag, 2006; Wang & Lee, 2009). Sarraf and McGuire (2020) utilized Chen's (2000) fuzzy TOPSIS method to rank the alternative paths. This fuzzy TOPSIS method is the most cited article in the TOPSIS application (Zyoud and Fuchs-Hanusch, 2017). In fuzzy TOPSIS, the rates of alternatives and the importance weight of each criterion are described by linguistic terms, which in turn they can be expressed in triangular fuzzy numbers. Next, a vertex method is defined to measure the distance between two triangular fuzzy numbers. The steps of Chen's (2000) fuzzy TOPSIS are as follows.

Step 1: The weight of criteria must be expressed in linguistic terms as shown in table 1.

Linguistic terms	Fuzzy triangular numbers
Very Low (VL)	(0, 0, 0.1)
Low (L)	(0, 0.1, 0.3)
Medium Low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Medium High (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1.0)
Very High (VH)	(0.9, 1.0, 1.0)

Table 1. Weights of criteria in fuzzy TOPSIS

Step 2: The linguistic ratings presented in table 2 are adopted to rate the alternatives with respect to each criterion.

Step 3: The linguistic terms are converted into triangular fuzzy numbers to construct the fuzzy decision matrix and the fuzzy weight of each criterion.

Step 4: Construct normalized fuzzy decision matrix R as shown in equation (6).

Linguistic terms	Fuzzy triangular numbers
Very Poor (VP)	(0, 0, 1)
Poor (P)	(0, 1, 3)
Medium Poor (MP)	(1, 3, 5)
Fair (F)	(3, 5, 7)
Medium Good (MG)	(5, 7, 9)
Good (G)	(7, 9, 10)
Very Good (VG)	(9, 10, 10)

Table 2. Linguistic ratings and associated fuzzy triangular numbers in fuzzy TOPSIS

$$R = [r_{ij}]_{m \times n}$$
$$r_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right), j \in B;$$
$$r_{ij} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right), j \in C; \quad (6)$$
$$c_j^* = \max_i c_{ij} \ \ if\ \ j \in B$$
$$a_j^- = \min_i a_{ij} \ \ if\ \ j \in C$$

where *B* is associated with the benefit, and *C* is associated with the cost criteria.

Step 5: Construct the weighted normalized fuzzy decision matrix V as shown in equation (7).

$$V = [v_{ij}]_{m \times n}$$
$$v_{ij} = r_{ij} \odot w_j \quad (7)$$

Step 6: Construct fuzzy positive-ideal solution (A^*) and fuzzy negative-ideal solution (A^-) as shown in equation (8).

$$A^* = (v_1^*, v_2^*, \dots, v_n^*)$$
$$A^- = (v_1^-, v_2^-, \dots, v_n^-) \quad (8)$$

where $v_j^* = (1,1,1)$ and $v_j^- = (0,0,0)$, j = 1, 2, …, n.

Step 7: Calculate the distance of each alternative from A^* and A^- using equation (9).

$$d_i^* = \sum_{j=1}^{n} d(v_{ij}, v_j^*), \qquad i = 1, 2, \dots, m.$$
$$d_i^- = \sum_{j=1}^{n} d(v_{ij}, v_j^-), \qquad i = 1, 2, \dots, m. \quad (9)$$

Where

$$d(m, n) = \sqrt{\tfrac{1}{3}[(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2]}$$

Step 8: Based on equation (10), calculate the closeness coefficient to determine the ranking order of alternatives.

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-}, i = 1, 2, \ldots, m. \qquad (10)$$

Step 9: Determine the ranking of alternatives according to the closeness coefficient.

5. INITIAL RESULTS OF MCDM METHODS

Figures 1 and 2 show five different paths from a source to a destination in Maryland with different travel times, travel distances, and safety levels. The safety level is calculated based on historical crash data on road segments. Road segments highlighted with blue color are considered safe, meaning that either there were no accidents on those road segments or there were a few accidents that can be ignored due to high traffic volume. Yellow indicates a medium level of safety, and red represents a high crash rate and high risk of an accident. The details of the safety level calculation are covered in (Sarraf & McGuire, 2020).

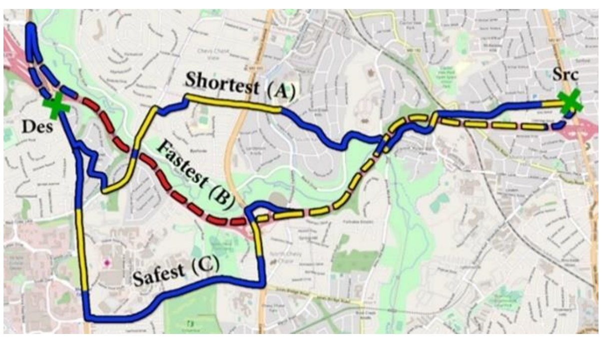

Figure 1. Shortest, fastest and safest paths from MD-97 (Src) to 1 Pooks Hill Road (Des)

Figure 2. Shortest path variants 1 and 2 from MD-97 (Src) to 1 Pooks Hill Road (Des)

The matrix shown in figure 3 presents these five alternative paths, the safety level, the distance, and the travel time for each path. Lower numbers in the matrix indicate more favorable results. For instance, the lowest number for the time criterion means the fastest path. Similarly, the lowest number for distance represents the shortest path and the lowest number for safety level indicates the safest path. For instance, the fastest path (alternative B) has a safety level of 115.99; this means this is an extremely high-risk path. The risk of driving on the fastest path (B) is 7.68 times more than driving on the shortest path (A). However, the fastest path is 1.58 times faster than the shortest path.

Paths	Safety	Distance	Time
Shortest (*A*)	15.1057	6.7356	508.0000
Fastest (*B*)	115.9982	7.6063	320.0000
Safest (*C*)	16.0389	8.2911	417.0000
Path 4 (*D*)	23.2725	6.7786	352.0000
Path 5 (*E*)	18.4128	6.9967	533.0000

Figure 3. Five alternative paths and their properties (first case study)

This matrix must be normalized before applying multi-criteria decision analysis to transform values into comparable values and to have the same range of values for each of the columns of the matrix. Except for TOPSIS and fuzzy TOPSIS that have their own normalization methods (equations (1) and (6)), for the remaining MCDM methods the normalization step is performed as follows: First, divide the elements of each column by the sum of the column as shown in equation (11). Next, the numbers must be changed in a way such that the higher number indicates a better choice; therefore, the multiplicative inverse of the elements is calculated as shown in equation (12), and then, the matrix is normalized again as shown in equation (13).

$$b_{ij} = \frac{a_{ij}}{\sum_{i=1}^{m} a_{ij}} \qquad (11)$$

$$c_{ij} = \frac{1}{b_{ij}} \qquad (12)$$

$$d_{ij} = \frac{c_{ij}}{\sum_{i=1}^{m} c_{ij}} \qquad (13)$$

Figure 4 illustrates the result of this transformation. In this matrix, higher numbers indicate more favorable results.

$$\begin{array}{cc} & \begin{array}{ccc}\text{Safety} & \text{Distance} & \text{Time}\end{array} \\ \begin{array}{l}\text{Shortest }(A)\\ \text{Fastest }(B)\\ \text{Safest }(C)\\ \text{Path 4 }(D)\\ \text{Path 5 }(E)\end{array} & \begin{bmatrix} 0.2824 & 0.2149 & 0.1612 \\ 0.0368 & 0.1903 & 0.2560 \\ 0.2659 & 0.1745 & 0.1964 \\ 0.1833 & 0.2135 & 0.2327 \\ 0.2316 & 0.2068 & 0.1537 \end{bmatrix}\end{array}$$

Figure 4. Final results of normalization for alternative paths (higher numbers indicate more favorable results).

The weights of criteria as shown in figure 5 were derived based on a user's input and the AHP method. The most influential factor for the user was the travel time (0.6434), followed by the safety level of the path (0.2828), and the least significant factor was the distance (0.0738). Similarly, as shown in figure 6, triangular fuzzy weights were obtained using fuzzy AHP and fed to fuzzy TOPSIS to determine the ranking of the paths. The detailed calculations for these weight calculations along with two other methods to derive fuzzy weights and feed them into fuzzy TOPSIS are presented in (Sarraf & McGuire, 2020).

$$\begin{array}{c} \begin{array}{ccc}\text{Safety} & \text{Distance} & \text{Time}\end{array} \\ \begin{bmatrix} 0.2828 & 0.0738 & 0.6434 \end{bmatrix}\end{array}$$

Figure 5. Vector of priorities (Criteria weights) obtained from AHP

$$\begin{array}{c} \begin{array}{ccc}\text{Safety} & \text{Distance} & \text{Time}\end{array} \\ \begin{bmatrix} (0.2423, 0.3692, 0.5543) & (0.1290, 0.1692, 0.2291) & (0.3189, 0.4615, 0.6652) \end{bmatrix}\end{array}$$

Figure 6. Triangular criteria weights obtained from fuzzy AHP

Table 3 presents the ranking results of the decision analysis methods and the weight assigned to each alternative path.

The matrix in figure 7 shows another example of alternative paths and the safety level, the distance, and the travel time for each alternative path.

$$\begin{array}{cc} & \begin{array}{ccc}\text{Safety} & \text{Distance} & \text{Time}\end{array} \\ \begin{array}{l}\text{Shortest }(A)\\ \text{Fastest }(B)\\ \text{Safest }(C)\\ \text{Path 4 }(D)\\ \text{Path 5 }(E)\end{array} & \begin{bmatrix} 181.7542 & 14.8009 & 715 \\ 181.7542 & 14.8009 & 715 \\ 90.0118 & 18.1377 & 1009 \\ 133.2662 & 16.5245 & 790 \\ 236.8785 & 17.4077 & 797 \end{bmatrix}\end{array}$$

Figure 7. Five alternative paths and their properties (second case study)

Figure 8 presents the transformed matrix. The shortest path and the fastest path are the same; therefore, the values in the first two rows of the matrix are identical. Path 5 (E) has the worst safety level, and its travel time is longer than alternatives A, B, and D.

$$\begin{array}{cc} & \begin{array}{ccc}\text{Safety} & \text{Distance} & \text{Time}\end{array} \\ \begin{array}{l}\text{Shortest }(A)\\ \text{Fastest }(B)\\ \text{Safest }(C)\\ \text{Path 4 }(D)\\ \text{Path 5 }(E)\end{array} & \begin{bmatrix} 0.1626 & 0.2192 & 0.2217 \\ 0.1626 & 0.2192 & 0.2217 \\ 0.3283 & 0.1789 & 0.1571 \\ 0.2217 & 0.1963 & 0.2006 \\ 0.1248 & 0.1864 & 0.1989 \end{bmatrix}\end{array}$$

Figure 8. Final results of normalization for alternative paths in the second use case (higher numbers indicate more favorable results).

Path	**AHP**	**Fuzzy AHP**	**TOPSIS**	**Fuzzy TOPSIS**	**PROMETHEE**
Shortest (A)	0.1994 (3)	0.2119 (3)	0.1962 (3)	0.1832 (2)	0.1999 (3)
Fastest (B)	0.1891 (4)	0.1644 (5)	0.1128 (5)	0.1766 (4)	0.1973 (4)
Safest (C)	0.2144 (2)	**0.2255** (1)	0.2372 (2)	0.1831 (3)	0.2036 (2)
Path 4 (D)	**0.2173** (1)	0.2121 (2)	**0.2691** (1)	**0.2834** (1)	**0.2043** (1)
Path 5 (E)	0.1796 (5)	0.1863 (4)	0.1848 (4)	0.1736 (5)	0.1949 (5)

Table 3. Comparison of Alternative Path Weights and Ranks Assigned in First Case Study.

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Path	AHP	Fuzzy AHP	TOPSIS	Fuzzy TOPSIS	PROMETHEE
Shortest (A)	0.2048 (3)	0.1970 (3)	0.2159 (2)	**0.2389** (1)	0.2012 (3)
Fastest (B)	0.2048 (3)	0.1970 (3)	0.2159 (2)	**0.2389** (1)	0.2012 (3)
Safest (C)	**0.2071** (1)	**0.2287** (1)	0.1772 (3)	0.1424 (4)	**0.2018** (1)
Path 4 (D)	0.2062 (2)	0.2094 (2)	**0.2518** (1)	0.2327 (2)	0.2016 (2)
Path 5 (E)	0.1770 (4)	0.1679 (4)	0.1394 (4)	0.1471 (3)	0.1943 (4)

Table 4. Comparison of Alternative Path Weights and Ranks Assigned in First Case Study

Table 4 presents the results of the decision analysis methods and the weights assigned to each alternative path in the second use case. Based on the criteria weights obtained from the user and our analysis, path 5 (alternative E) must be considered as the last suggested path. The ranking results of AHP and PROMETHEE are considered as the best-ranking result in this case study.

All the decision-making methods except Fuzzy TOPSIS considered path 5 as the last suggested path. In addition, TOPSIS considered the safest path (alternative C) as the third-best path, while AHP and PROMETHEE ranked the safest path as the best alternative.

6. IMPROVING THE RESULTS OF TOPSIS AND FUZZY TOPSIS

AHP and PROMETHEE produced the best ranking results in both of the case studies. In the second case study, the safest path is selected as the first alternative path by AHP, fuzzy AHP, and PRROMETHEE, while TOPSIS ranks it as the third alternative path and fuzzy TOPSIS ranks it as the fourth alternative path. Based on the results of the use cases, inconsistent results of TOPSIS and fuzzy TOPSIS were noticed. This section aims to improve the results of TOPSIS and fuzzy TOPSIS by presenting a few changes.

In contrast with the literature that suggests vector normalization is among the most suitable methods for TOPSIS, we noticed that the evaluation results of TOPSIS in the case studies are affected by the vector normalization method that is presented in equation (1). Since the initial values of the safety level and time of the alternative paths can vary significantly, it is more suitable to use the normalization method presented in equations (11), (12), and (13).

Table 5 presents the safety level of the first case study, which was presented in figure 3, along with the normalized data obtained from the TOPSIS method and the normalization method we employed. The table also includes the skewness of each dataset. Skewness is a measure to determine if the data is symmetrically distributed. A value of zero shows a perfectly symmetrical distribution. A positive value shows positive skewness and a negative value shows a negative skewness.

	Safety Level	TOPSIS Normalization	Our Normalization
Path A	15.1057	0.1241	0.2824
Path B	115.9982	0.9528	0.0368
Path C	16.0389	0.1317	0.2659
Path D	23.2725	0.1912	0.1833
Path E	18.4128	0.1512	0.2316
Skewness	1.32	1.32	-0.9

Table 5. The skewness of Data in the First Case Study

The skewness of the data can be the cause of the inconsistent ranking results of TOPSIS for the first case study. The initial data is positively skewed. Similarly, TOPSIS vector normalization results in identical skewness as the initial data. Osborne (2002) indicates that multiplicative inverse is one of the most commonly discussed methods to reduce skewness. Our normalization method employs multiplicative inverse; therefore, as show in table 5, it results in the skewness of -0.9. This normalization method improved the normality of the data and inversed the results; consequently, it is now negatively skewed and the skewness result is closer to zero.

Table 6 and Table 7 present the results and ranking of AHP, initial TOPSIS, and TOPSIS with reciprocal normalization method for the first and second case studies, respectively. Clearly, the normalization method had a huge impact on the results. The TOPSIS method with reciprocal normalization produced identical ranking results to AHP and PROMETHEE in both case studies.

Path	**AHP**	**TOPSIS**	**TOPSIS-Reciprocal Normalization**
Shortest (A)	0.1994 (3)	0.1962 (3)	0.1912 (3)
Fastest (B)	0.1891 (4)	0.1128 (5)	0.1745 (4)
Safest (C)	0.2144 (2)	0.2372 (2)	0.2313 (2)
Path 4 (D)	**0.2173 (1)**	**0.2691 (1)**	**0.2416 (1)**
Path 5 (E)	0.1796 (5)	0.1848 (4)	0.1614 (5)

Table 6. Results and Ranking of the First Case Study Using TOPSIS with Reciprocal Normalization

Path	**AHP**	**TOPSIS**	**TOPSIS-Reciprocal Normalization**
Shortest /fastest (A/B)	0.2048 (3)	0.2159 (2)	0.1989 (3)
Safest (C)	**0.2071 (1)**	0.1772 (3)	**0.2439 (1)**
Path 4 (D)	0.2062 (2)	**0.2518 (1)**	0.2289 (2)
Path 5 (E)	0.177 (4)	0.1394 (4)	0.1295 (4)

Table 7. Results and Ranking of the Second Case Study Using TOPSIS with Reciprocal Normalization

In addition, the results of fuzzy TOPSIS can be improved. This requires the following two steps: To rate alternatives using the linguistic terms, first the difference of the lowest and the highest values for each individual column were calculated. Next, the range was divided into seven equal parts and the alternatives were rated based on the linguistic terms presented in Table 2. However, this method isolates the results of each criterion. Thus, a good or bad result in one criterion cannot directly influence the results of other criteria. For instance, if the safety level of alternative A is very good (VG) compared to the other alternatives, and the time of alternative B is very good (VG) compared to other alternatives, we cannot find the ratio between very good safety level and very good travel time. The alternative path matrix is normalized; therefore, all the criteria, namely safety level, time, and distance are comparable. In order to fix this issue, it is necessary to calculate the difference of the global lowest and the global highest values among all the benefit criteria, and divide the range into seven equal parts and rate the alternatives based on the linguistic terms. A similar approach must be repeated for all the cost criteria.

Figure 9 shows the linguistic terms assigned to the alternative results that were presented in figure 3 using local lowest and local highest values in each individual column. For instance, the lowest value in the distance column is 0.1745 and the highest is 0.2149. By calculating the difference between these two values and dividing the range into seven equal parts, the linguistic terms were assigned. Figure 10 presents the modified rating of the alternatives in linguistic terms using the global lowest and global highest in the entire matrix. The global lowest value in the matrix is 0.0368 and the global highest value is 0.2824. Using the global difference, the alternatives are rated based on the linguistic terms.

$$\begin{array}{l} \\ \text{Path A} \\ \text{Path B} \\ \text{Path C} \\ \text{Path D} \\ \text{Path E} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Safety} & \text{Distance} & \text{Time} \end{array} \\ \begin{bmatrix} VG & VG & VP \\ VP & MP & VG \\ VG & VP & MP \\ MG & VG & G \\ G & G & VP \end{bmatrix} \end{array}$$

Figure 9. Alternative paths and their properties in linguistic terms using local lowest and highest values.

$$\begin{array}{l} \\ \text{Path A} \\ \text{Path B} \\ \text{Path C} \\ \text{Path D} \\ \text{Path E} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Safety} & \text{Distance} & \text{Time} \end{array} \\ \begin{bmatrix} VG & G & F \\ VP & MG & VG \\ VG & F & MG \\ MG & G & G \\ G & MG & F \end{bmatrix} \end{array}$$

Figure 10. Alternative paths and their properties in linguistic terms using global lowest and highest values.

The next step to improve the accuracy of fuzzy TOPSIS is changing its initial normalization step that is shown in equation (6) and shown again for convenience in equation (14). In this equation, c_j^* is calculated for each individual benefit criterion; this is the maximum value of each benefit column. This needs to be changed to c^* which is the global maximum c among a set of benefit criteria. Similarly, a_j^- must be changed to a^- which is the global minimum a among a set of cost criteria. Moreover, r_{ij} must be calculated based on global c^* and a^-. Equation (15) presents the modified version of the normalized fuzzy decision matrix for fuzzy TOPSIS.

$$
\begin{aligned}
&R = [r_{ij}]_{m\times n} \\
&r_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right), j \in B; \\
&r_{ij} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right), j \in C; \\
&c_j^* = \max_i c_{ij} \;\; if \;\; j \in B \\
&a_j^- = \min_i a_{ij} \;\; if \;\; j \in C
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
&R = [r_{ij}]_{m\times n} \\
&r_{ij} = \left(\frac{a_{ij}}{c^*}, \frac{b_{ij}}{c^*}, \frac{c_{ij}}{c^*}\right), j \in B; \\
&r_{ij} = \left(\frac{a^-}{c_{ij}}, \frac{a^-}{b_{ij}}, \frac{a^-}{a_{ij}}\right), j \in C; \\
&c^* = max\, c_{ij} \;\; if \;\; j \in B \\
&a^- = min\, a_{ij} \;\; if \;\; j \in C
\end{aligned}
\tag{15}
$$

Applying the aforementioned steps to fuzzy TOPSIS improves the results of fuzzy TOPSIS. Table 8 and Table 9 present the results and ranking of alternative paths using AHP and fuzzy TOPSIS for the first and the second case studies, respectively. The ranking result of improved fuzzy TOPSIS for the first case study is closer to the result of AHP. In the second case study, fuzzy TOPSIS initially produced the worst results. However, the improved fuzzy TOPSIS enhanced the ranking results of fuzzy TOPSIS significantly and now it is identical to the ranking of AHP.

Path	AHP	Fuzzy TOPSIS	Improved Fuzzy
Shortest (A)	0.1994 (3)	0.1832 (2)	0.2060 (3)
Fastest (B)	0.1891 (4)	0.1766 (4)	0.1591 (5)
Safest (C)	0.2144 (2)	0.1831 (3)	0.2141 (2)
Path 4 (D)	**0.2173 (1)**	**0.2834 (1)**	**0.2261 (1)**
Path 5 (E)	0.1796 (5)	0.1736 (5)	0.1974 (4)

Table 8. Results and Ranking of the First Case Study Using Improved Fuzzy TOPSIS

Path	AHP	Fuzzy TOPSIS	Improved Fuzzy
Shortest (A)	0.2048 (3)	**0.2389 (1)**	0.2097 (3)
Fastest (B)	0.2048 (3)	**0.2389 (1)**	0.2097 (3)
Safest (C)	**0.2071 (1)**	0.1424 (4)	**0.2324 (1)**
Path 4 (D)	0.2062 (2)	0.2327 (2)	0.2184 (2)
Path 5 (E)	0.1770 (4)	0.1471 (3)	0.1299 (4)

Table 9. Results and Ranking of the Second Case Study Using Improved Fuzzy TOPSIS.

7. CONCLUSIONS

Based on two real-world case studies and a comparison of five multi-criteria decision-making methods we realized AHP and PROMETHEE produce the best ranking results, whereas TOPSIS and fuzzy TOPSIS produce inconsistent results compared to other MCDM. After evaluating TOPSIS and fuzzy TOPSIS, we concluded that the normalization methods are the source of the inconsistent results. Although multiple research studies suggested the use of vector normalization in TOPSIS, we concluded that the normalization method presented in this research study that is based on multiplicative inverse is more appropriate for our use cases. This normalization method resulted in producing identical ranking results to AHP in both case studies.

Furthermore, the source of poor results in fuzzy TOPSIS is due to assigning linguistic terms to the values in each individual column and the normalization step that is performed individually for each column. We could improve the ranking results of fuzzy TOPSIS. This requires two steps: first finding the difference of the global lowest and highest values of all the benefit criteria and assigning the linguistic terms based on the difference that is divided into seven equal parts. Next, changing the normalization step to perform normalization based on all the columns of the same type, not individual columns. These changes significantly improved the results of fuzzy TOPSIS since the result is now identical to the ranking results of AHP.

A future work will be to gather the data and the ranking results of TOPSIS and fuzzy TOPSIS in the literature and apply the aforementioned changes and compare the new ranking results with the results already presented in the literature. We believe the changes presented in this research study are simple yet effective to improve the ranking results of TOPSIS and fuzzy TOPSIS.

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