# **Two Frameworks for Discrete Mathematics**

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## **ABSTRACT**

Information Systems and Computer Science educators have campaigned to increase mathematical content in the computing curriculum. Unfortunately, mathematical concepts are often presented in a manner that conflicts with the general mental framework, or gestalt, of IS and CS students. However, there is more than one gestalt in mathematics. In previous research, we developed two scales for measuring mathematical gestalt in books--a Logical Math scale and a Computational Math scale. In this paper, we apply our two scales to current Discrete Math textbooks to assess the relative emphasis these books give to each gestalt. Our findings have relevance in the development of approaches for teaching mathematical topics in computer courses, especially Discrete Math courses.

**Keywords:** framework, gestalt, discrete math, logical math, computational math.

# 1. INTRODUCTION

Information Systems and Computer Science educators have campaigned to increase mathematical content in the computing curriculum. Reasons for this zeal include faith in the positive effects of mathematics on the mind (Ralston, 2005), as well as the belief that mathematical logic provides skills that improve the software development process (Kim, 2003). If one agrees with this position, then one must decide how mathematical topics should be presented to IS and CS students. In an ideal world, mathematical ideas would blend naturally into the general mental framework, or *gestalt*, of these students.

In *The Mathematical Experience*, Davis and Hersh (1981) state: "People vary dramatically in what might be called their *cognitive style*, that is, their primary mode of thinking." Unfortunately, the traditional gestalt of mathematics often conflicts with the cognitive style of many IS and CS students. Davis and Hersh observe that:

The definition-theorem-proof approach to mathematics has become almost the sole *paradigm* of mathematical exposition and advanced instruction.

However, there is more than one gestalt in mathematics. Some authors have not succumbed to a purely logical view of mathematics. In their book *What is Mathematics?* Courant and Robbins (1941) offer a constructive approach:

There seems to be a great danger in the prevailing overemphasis on the deductive-postulational character of mathematics. The element of constructive invention, of directing and motivating intuition, remains the core of any mathematical achievement, even in the most abstract fields.

The classic proponent of a problem-solving approach to mathematics was Polya (1945) in his book *How to Solve It*:

Studying the methods of solving problems, we perceive another face of mathematics. Yes, mathematics has two faces; it is the rigorous science of Euclid, but it is also something else. Mathematics presented in the Euclidean way appears as a systematic, deductive science; but mathematics in the making appears as an experimental, inductive science. Both aspects are as old as the science of mathematics itself.

In an earlier paper (McMaster, 2007a), we describe our efforts to characterize and measure two gestalts for mathematics, one based on proving theorems and the other based on solving problems. Our methodology makes the assumption that words used frequently in a book indicate the gestalt of the author. By comparing word frequencies in various mathematics books, we developed two scales for measuring the mathematical gestalt of the authors. A Logical Math scale measures theorem-proving gestalt, and a Computational Math scale measures problem-solving gestalt. We use the term "computational" in the broad sense described by the Program in Applied and Computational Mathematics at Princeton University (www.pacm.princeton.edu).

# 2. RESEARCH PROBLEM

The prevailing view of mathematics in Discrete Math courses seems to favor theorems and proofs. Koshy (2004) claims in his Discrete Math book that "Theorems are the backbones of mathematics." Gossett's (2002) textbook entitled *Discrete Math with Proof* advertises that proofs are "inside the book", like a nutritional supplement. Gossett's book is not the only one with the word "proof" in the title.

With respect to mathematical gestalt, some Discrete Math books have split personalities. Several have the word "Applications" in the title. Kolman (2007) expects students "to develop...the ability to write a mathematical proof", but also includes computational experiments at the end of most chapters.

Gersting (2006) stresses "the importance of logical thinking, the power of mathematical notation, and the usefulness of abstractions", but her book includes more computer applications than most Discrete Math texts.

In this paper, we apply our Logical Math and Computational Math scales to a sample of 25 current Discrete Math textbooks. Our primary purpose is to assess the level of emphasis these books give to each gestalt. Our findings have relevance in the development of approaches for teaching mathematical topics in computer courses, especially Discrete Math courses.

## 3. MATH GESTALT SCALES

The methodology we used to create our Logical Math and Computational Math gestalt scales is summarized in this section. A more detailed description is presented in our earlier paper (McMaster, 2007a). The methodology involved the following steps.

Sampling. Choose a broad sample of books from many mathematical areas. We selected 112 books from the Amazon web site that included a *concordance* (a list of frequently used words). We made an *a priori* classification of each book as either Traditional Math or Applied Math based on title and content.

Data Collection. The Amazon concordance for a book provides a list of the 100 most frequently used words. An Amazon concordance screens out many common English words, such as "the" and "of". For each concordance word, we recorded the book, word, and frequency (total number of times the word occurs in the book).

Transform Frequencies. We transformed word frequencies within each concordance because books vary in their total number of words. We excluded any Top 100 Common English Words (Fry, 1993) that had not already been removed by Amazon. For the remaining words, we calculated the average word frequency per book. We then divided each word frequency by the average and multiplied by 100, giving a "standard frequency" (StdFreq). This sets the average word frequency to 100.

Convert and Combine Words. We converted many words to a standard form, so that the scale contribution of a word would not depend on the particular form or tense an author favored. For example, we converted plural nouns to singular form and converted verbs to present tense. In selected cases, we combined two or more synonyms into one word (e.g. "theorem" and "lemma" became "theorem/lemma").

Construct Gestalt Scales. Constructing the Logical Math scale (referred to as LMATH) and the Computational Math scale (referred to as CMATH) was an iterative process. We searched for words that are used *frequently* within each book, and *consistently* across similar books. During each iteration:

- Query all Traditional Math books for the LMATH scale. Query all Applied Math books for the CMATH scale. Find all words in which the average StdFreq taken across all books for that scale is relatively "high".
- Calculate a weight for each of the chosen words.
- Calculate the LMATH and CMATH scores for all books in the sample. Each score is a weighted average of the StdFreq values for words on the scale.
- 4. Remove books from the sample that have a relatively low score on their relevant scale.
- 5. Repeat the above steps, obtaining a revised list of words and weights for each scale. Stop when the scale words and weights do not change.

After several iterations, we obtained LMATH and CMATH scales constructed from 25 Traditional Math books and 25 Applied Math books, respectively. Each gestalt scale consists of a list of word/groups and weights. A word/group is created when two or more word forms (e.g. "proof" and "prove") or synonyms (e.g. "function" and "map") are combined.

## **Logical Math Gestalt**

The Logical Math (LMATH) scale consists of 10 word/groups and weights. The details of this scale are presented in Table 1. The most frequent word/group for the LMATH scale is "theorem/lemma/corollary". As expected, the concepts of "theorem", "proof", and "definition" (with equivalent words combined) are represented on the scale. Surprisingly, the most frequent individual word is "let". Words like "hence/thus/therefore", "show", "follow", and "since" are conventions commonly used to convey logical ordering in proofs.

**Table 1: LOGICAL MATH Words**Based on 25 Traditional Math books

Word/Group	Books	Avg StdFreq	Weight
theorem/lemma /corollary	25	428.2	18.34
let	25	418.1	17.78
set/subset	25	333.1	13.03
proof/prove	25	329.9	12.85
function/map	25	300.9	11.23
hence/thus/ therefore	25	230.3	7.28
definition/define	25	221.4	6.78
show/shown	25	191.3	5.10
follow/following	24	175.4	4.21
since	24	160.8	3.40
TOTAL			100.00

The words "function" and "set" appear on the scale because these terms are commonly used throughout mathematics. Words specific to only a few areas of mathematics (e.g. "derivative" and "ring") do not appear on the scale. Thus, LMATH measures a general gestalt for Traditional Mathematics.

# **Computational Math Gestalt**

The Computational Math (CMATH) scale consists of 10 word/groups and weights. The details of this scale are presented in Table 2. The most frequent word for the CMATH scale is "problem", and the third most frequent word is "solution/solve". This confirms that problem solving is a central theme in the Computational Math gestalt of Applied Math books.

**Table 2: COMPUTATIONAL MATH Words**Based on 25 Applied Math books

Word/Group	Books	Avg StdFreq	Weight	
problem	25	394.3	18.56	
method/algorithm	23	343.1	15.33	
solution/solve	25	322.4	14.03	
value/variable	24	271.7	10.83	
equation/inequality	21	265.9	10.46	
function/map	24	257.0	9.90	
model/modeling	19	222.3	7.71	
point/line	24	175.8	4.78	
system/subsystem	23	168.7	4.33	
condition/constraint	25	164.3	4.06	
TOTAL			100.00	

CMATH gestalt is concerned with how to use mathematics to solve real-world problems. The words "model" and "method/algorithm" describe the main approach to solving problems. Words like "variable", "equation",

"function", and "condition/constraint" are components of mathematical models and algorithms. The word "model" is an essential part of Computational Math gestalt. By comparison, "model" rarely appears in Traditional Math books. In Logical Math gestalt, the real world is irrelevant, so there is no need for models.

The word "function" appears on both the LMATH and the CMATH scales, although with slightly different weights. We considered excluding this word from both scales to make the scales more "orthogonal," but decided to retain "function" on each scale.

## **LMATH Calculations**

We can learn a lot about a book from the calculation of its LMATH (and CMATH) scores. The calculations show which words contribute most to the overall gestalt score for a book. Royden's (1988) *Real Analysis* was one of the 25 Traditional Math books used to develop the final LMATH scale. The LMATH calculations for this book are shown in Table 3.

**Table 3: LMATH Calculations** Royden -- Real Analysis

Word/Group	Weight	Std Freq	LMATH Scale
theorem/lemma/ corollary	18.34	214.8	39.4
let	17.78	413.1	73.4
set/subset	13.03	962.3	125.4
proof/prove	12.85	188.7	24.2
function/map	11.23	527.0	59.2
hence/thus/ therefore	7.28	226.8	16.5
definition/define	6.78	230.0	15.6
show/shown	5.10	237.4	12.1
follow/following	4.21	127.7	5.4
since	3.40	187.3	6.4
TOTAL			377.6

Royden's concordance includes all 10 of the LMATH scale word/groups. The StdFreq value indicates how often a scale word is used in the book. The weight defines the importance of a word for measuring gestalt (as derived in Table 1). For Royden, the most frequent words are "set/subset" (StdFreq = 962.3), "function/map" (StdFreq = 527.0), and "let" (StdFreq = 413.1). The LMATH value of 377.6 can be interpreted as follows: scale words appear (on average) about 3.776 times more often than an average concordance word in the same

book. Thus, Royden's book exhibits a high level of Logical Math gestalt.

## **CMATH Calculations**

The field of Operations Research can be viewed as an archetype for Computational Math gestalt. The Operations Research titles in our original sample of Applied Math books consistently scored high on the CMATH scale. We performed CMATH calculations for Hillier & Lieberman's (2004) *Operations Research* textbook, resulting in a CMATH score of 303.7. The CMATH results for this book are shown in Table 4.

**Table 4: CMATH Calculations**Hillier & Lieberman -- Operations Research

Word/Group	Weight	Std Freq	CMATH Scale
problem	18.56	483.8	89.8
method/algorithm	15.33	262.7	40.3
solution/solve	14.03	496.9	69.7
value/variable	alue/variable 10.83 47		51.4
equation/inequality	10.46	69.3	7.2
function/map	9.90	114.4	11.3
model/modeling	7.71	268.3	20.7
point/line	4.78		
system/subsystem	4.33	137.4	5.9
condition/constraint	4.06	180.7	7.3
TOTAL			303.7

Hillier & Lieberman's concordance includes 9 of the 10 CMATH scale word/groups. The most frequent words are "solution/solve" (StdFreq = 496.9), "problem" (StdFreq = 483.8), and "value/variable" (StdFreq = 474.4). Also, both "model" and "method/algorithm" have StdFreq values above 250.

# 4. DISCRETE MATH FRAMEWORKS

Two related questions concern mathematical gestalt for Discrete Math.

- Which gestalt is emphasized in Discrete Math textbooks?
- 2. Which gestalt is more suitable for Discrete Math courses?

This paper focuses primarily on the first question. We wanted to determine how much support current Discrete Math textbooks provide for each framework. The answer to this question can influence how teachers respond to the second question.

In this study, we selected a sample of 25 Discrete Math books having Amazon concordance data. Unfortunately, concordance data

was not available for several well-known Discrete Math textbooks. We transformed frequencies and converted/combined words as in our scale development methodology. Then we calculated Logical Math and Computational Math gestalt measures for each book using the LMATH and CMATH scales defined previously. The scale scores for the 25 books are listed in Appendix 1.

## **Gestalt Words in Discrete Math Books**

Table 5 displays eight word/groups selected from the two gestalt scales. Each word is presented with the number of Discrete Math books that use the word frequently, along with the average standard frequency (Avg StdFreq) value. The Logical Math words "theorem", "proof", and "set" were used frequently in at least 22 of the books. The word "problem" was the only Computational Math word used frequently by as many as 18 books.

**Table 5: Gestalt Scale Words** 25 Discrete Mathematics books

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Word/Group	Books	Avg StdFreq
Logical Math		
set/subset	25	427.0
proof/prove	22	217.0
definition/define	17	210.6
theorem/lemma/corollary	23	180.3
Computational Math		
method/algorithm	17	145.3
solution/solve	14	139.2
problem	18	127.4
model/modeling	1	52.8

The average StdFreq values are much higher for the Logical Math words. This indicates that Discrete Math books contain more Logical Math words than Computational Math words. It is disappointing that only one Discrete Math book listed "model" as a frequent word. According to Kramer (2007), "modeling is the most important engineering technique."

## **LMATH Scores for Discrete Math Books**

For the 25 Discrete Math books, the LMATH scores ranged from a minimum of 94.3 to a maximum of 348.9, with a mean of 200.3. The distribution of LMATH scores is presented in Figure 1. In this graph, scores are grouped into intervals of size 50.

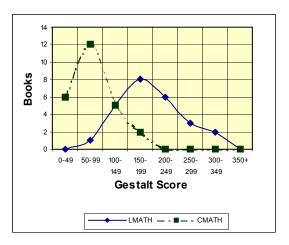


Figure 1: Mathematical Gestalt Scores
25 Discrete Math Books

Eleven of the 25 books had LMATH scores above 200. A LMATH score of 200 means that scale words appear twice as often as the "average concordance word." Two of the books had LMATH scores above 300: (1) Nievergelt's (2002) Foundations of Logic and Mathematics (LMATH = 348.9), and (2) Gries' (1993) A Logical Approach to Discrete Math (LMATH = 316.5). Note that these books contain "logic" in the title.

The LMATH calculations for Gries' textbook are shown in Table 6. The word/groups with StdFreq values above 300 are "proof/prove" "theorem/lemma/corollary", "set/subset", and "definition/define". Gries' book is vintage Logical Math.

Table 6: LMATH Calculations

Gries -- A Logical Approach to Discrete Math

Word/Group	Weight	Std Freq	LMATH Scale
theorem/lemma/ corollary	18.34	422.8	77.5
let	17.78	103.0	18.3
set/subset	13.03	411.0	53.6
proof/prove	12.85	716.3	92.0
function/map	11.23	192.9	21.7
hence/thus/ therefore	7.28	164.8	12.0
definition/define	6.78	342.7	23.2
show/shown	5.10	101.1	5.2
follow/following	4.21	220.0	9.3
since	3.40	108.6	3.7
TOTAL			316.5

# **CMATH Scores for Discrete Math Books**

The 25 CMATH scores ranged from 36.8 to 175.2, with a mean of 83.8. The distribution of CMATH scores, compared to LMATH scores, is shown in Figure 1. The excess of lower CMATH scores is evident in this graph.

No Discrete Math book in the sample had a CMATH score above 200. The two books with highest CMATH scores are: (1) Rosen's (1999) Handbook of Discrete and Combinatorial Math (CMATH = 175.2), and (2) Alt's (2001) Computational Discrete Mathematics (CMATH = 161.1). We note that, even though Alt's book has the word "computational" in the title, the LMATH score (181.8) for this book is higher than its CMATH score.

The CMATH calculations for Rosen's Handbook (which is not his popular Discrete Mathematics and It's Applications textbook) are shown in Table 7. This book promotes "methods/algorithms" but with little emphasis on "models". However, this was the only Discrete Math book in the sample to include "model" in its concordance.

**Table 7: CMATH Calculations**Rosen -- Handbook of Discrete and Combinatorial Math

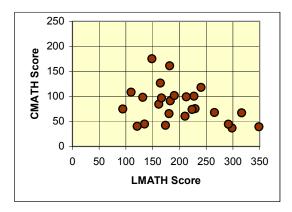
Word/Group	Weight	Std Freq	CMATH Scale
problem	18.56	206.5	38.3
method/algorithm	15.33	341.3	52.3
solution/solve	14.03	105.4	14.8
value/variable	10.83	182.1	19.7
equation/inequality	10.46	55.3	5.8
function/map	9.90	241.3	23.9
model/modeling	7.71	52.8	4.1
point/line	4.78	249.6	11.9
system/subsystem	4.33	101.3	4.4
condition/constraint	4.06		
TOTAL			175.2

## **LMATH vs. CMATH Scores**

For 24 of the 25 Discrete Math books, the LMATH score is higher than the CMATH score. The exception was Rosen's (1999) Handbook of Discrete and Combinatorial Math. Logical Math gestalt is consistently the predominant framework in Discrete Math books, even when the authors declare they are providing balanced coverage of theorem proving and problem solving.

The relationship between LMATH and CMATH scores for the 25 Discrete Math books is presented as a scatter diagram in Figure 2.

For descriptive purposes, the (negative) correlation between LMATH and CMATH scores is -0.343. Books with high LMATH scores tend to have low CMATH scores. The converse is not true, because no book in the sample had a high CMATH score.



**Figure 2: LMATH vs. CMATH Scores** 25 Discrete Math Books

## 5. SUMMARY AND CONCLUSIONS

The main purpose of this study was to measure two mental frameworks, or gestalts, in Discrete Math books. Logical Math gestalt is based on proving theorems, while Computational Math gestalt is based on solving problems. The gestalt scales are based on words that are used frequently in mathematics books. Our Logical Math (LMATH) scale contains 10 word/groups, including theorem/lemma, proof, let, and definition. The Computational Math (CMATH) scale has 10 word/groups, including problem, solution, model, and method/algorithm.

We calculated LMATH and CMATH scores for 25 Discrete Math books. For 24 of the books, the LMATH score is higher than the CMATH score, indicating an emphasis on Logical Math gestalt. Of particular note, only one Discrete Math book included "model" as a frequent word. The data also showed a slight negative relationship between LMATH and CMATH scores, primarily because the highest LMATH books had low CMATH scores. However, no Discrete Math book had a high CMATH score.

## Which Framework for Discrete Math?

Our intent in this study was not to determine which framework is "best" for Discrete Math, since each gestalt can provide value in the course. The Realistic Math Education program in Holland, based on the work of Treffers (1987), encourages two types of "mathematization":

- Horizontal mathematization, where students solve a problem located in a real-life situation (this is Computational Math).
- Vertical mathematization, where students reorganize concepts within the mathematical system itself (this is Logical Math).

Both types of mathematization encourage abstraction--by constructing models for real-world systems, and through manipulating symbolic objects in proofs.

# Computational Math and Software Development

Logical Math and Computational Math are not the only mental frameworks relevant to Information Systems and Computer Science. We are currently developing gestalt measures for software development, with interrelated scales for programming, database, and software engineering. In our initial sample of software development books, frequent programming words include class and method; frequent database words include data and database; and frequent software engineering words include software and system. Computational Math gestalt, where models and algorithms are used to solve real-world problems, appears to be closer in spirit to the framework used in software development.

We are continuing our efforts to combine mathematical and software frameworks in Information Systems and Computer Science courses. In his Discrete Math book, Koshy (2004) admits that "knowledge of a structured programming language, such as Java or C++, can make the study of discrete mathematics more rewarding." If a Discrete Math course is taught with a Computational Math gestalt, then it is natural to include programming in the course (see McMaster, 2007b). In computing, models and algorithms are implemented as software.

Each Discrete Math teacher should choose an appropriate blend of Logical Math and Computational Math for the course. If the instructor wants to emphasize Logical Math, then she/he can consider the 11 Discrete Math books with LMATH scores above 200. If a Computational Math emphasis is desired, then the instructor has a dilemma. No Discrete Math book in our sample promotes this framework. Discrete Math books not included in our sample could be examined, with special attention paid to their use of CMATH scale words such as "model" and "algorithm". Or the instructor can prepare custom classroom materials and assignments that are based on Computational Math gestalt.

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**APPENDIX 1: 25 Discrete Mathematics Books** 

Author	Year	Title	LMATH	СМАТН
Aigner	2007	Discrete Mathematics	240.6	118.0
Alt	2001	Computational Discrete Math	181.8	161.1
Anderson	2005	First Course in Discrete Math	134.8	44.9
Biggs	2002	Discrete Mathematics	227.3	100.5
Chetwynd	1995	Discrete Mathematics	180.6	65.6
Ensley	2005	Discrete Math: Math Reasoning and Proof	183.1	91.0
Foldes	1994	Fund Structures of Algebra and DM	298.5	36.8
Garnier	1992	Discrete Math For New Technology	229.5	75.5
Gries	1993	Logical Approach to Discrete Math	316.5	67.2
Laywine	1998	Discrete Math Using Latin Squares	121.1	40.4
Lipschutz	1991	2000 Solved Problems in Discrete Math	265.5	67.9
Lipschutz	1997	Schaum Outline of Discrete Math	223.8	73.7
Lovasz	2003	Discrete Mathematics	174.2	42.3
Molluzzo	1997	First Course in Discrete Mathematics	164.5	126.6
Nievergelt	2002	Foundations of Logic and Math	348.9	39.2
O'Donnell	2006	Discrete Math Using a Computer 2	213.0	99.1
Pemmaraju	2003	Computational Discrete Mathematics	94.3	74.7
Penner	1999	DM: Proof Techniques and Math Structures	291.6	44.3
Piff	1991	Discrete Mathematics	210.5	60.5
Rosen	2006	Discrete Mathematics	161.4	84.5
Rosen	1999	Handbook of Discrete Math and Comp Math	148.6	175.2
Ross	2000	Finite and Discrete Mathematics	166.7	96.8
Sethumadhavan	2006	Discrete Mathematics	190.6	101.6
Wallis	2002	Beginner's Guide to Discrete Math	131.5	98.2
Wazwaz	2004	Discrete Mathematics	109.8	108.4